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Electromagnetic fields of a helix, and related topics

In the regime of interest for heavy ion acceleration, where the wave speeds are much less than the speed of light and the transverse dimensions are a small fraction of a free space wavelength, quasi-static approximations to the EM fields should be sufficient.

One "tricky" aspect involved in developing these quasi-static field solutions is how to define "voltage" when curl (E) isn't zero. It is also important to recognize that the dominant source for the electric field is actually the charge on the helix wires and not the time derivative of the magnetic flux.

The essential approximation involved is a very small pitch on the helix, namely

$$\tan\Psi = \frac{s}{2\pi a} \ll 1 \tag{1}$$

where $s = 1/n$ is the wire spacing. A "smooth" approximation to the currents and charges on the helix can then be used to calculate the fields at distances of order s/2 away from the wires (essentially a sheath helix model). A current $I(z,t)$ flowing in the helical wires is then equivalent to an azimuthal sheet current

$$K_\theta = \frac{I}{s} \tag{2}$$

that is much larger than the axial sheet current $K_z = I/2\pi a$. As a consequence, the dominant magnetic field components are B_z and B_r created by this azimuthal current; B_θ is smaller by a factor of $s/2\pi a$ (1% in the oil helix).

An azimuthally symmetric charge per unit length λ (equivalent to a surface charge $\sigma = \lambda/2\pi a$ on the sheath helix) is related to the current in the helical wires by the continuity equation

$$\frac{\partial\lambda}{\partial t} = -\frac{\partial I}{\partial z} \tag{3}$$

This charge acts as a source for an azimuthally symmetric electric field with components E_r, E_z. If we neglect $\partial B_\theta/\partial t$ compared to this charge as the dominant source of the electric field, in a plane with $\theta = const$ we can use

$$\vec{E} = -\nabla\phi \tag{4}$$

to describe the quasi-static electric field, and define the voltage on the helix as

$$V(z,t) = \phi(r = a, z, t) \tag{5}$$

The final "link" required is the connection between this helix voltage and the time changing axial magnetic flux inside the helix. Using a path integral in Faraday's law that goes <u>inside</u> the helix wires at their radius $r = a$ over a distance Δz, encircling the magnetic flux inside the helix $n\Delta z$ times, we have a voltage change over Δz given by

$$\Delta V = -n\,\Delta z \int_0^a \frac{\partial B_z(r,z,t)}{\partial t} 2\pi r dr \tag{6}$$

In the continuous limit, we have

$$\frac{\partial V(z,t)}{\partial z} = -n \frac{\partial \Phi(z,t)}{\partial t} \tag{7}$$

Here

$$\Phi(z,t) = \int_0^a B_z(r,z,t)\, 2\pi r dr \tag{8}$$

is the <u>total</u> flux inside the helix, created by both the helix azimuthal current and the primary strap current (when transformer coupling is used).

Note that the azimuthal electric field at the helix $E_\theta = -\frac{1}{2\pi a} \partial\Phi / \partial t$ is $s / 2\pi a$ smaller than the axial electric field (order of 1%).

We also emphasize that the "sheath model" of the helix describes the EM fields accurately outside of radial distances within s/2 of the helix (~ 2 mm), since the spatial harmonics needed in a full field solution decay as $\exp -\frac{2\pi m}{s}|r-a|$. The great increase in complexity involved in resolving the EM fields on the fine scale of the wires seems unnecessary, except for quantifying the stresses in that region.

Computational and analytical field models:

I'm not qualified to judge the best way to formulate a computational model, but it would seem that electrostatic and magnetostatic field solutions, with

$$\nabla^2 \phi = 0, \quad E = -\nabla \phi,$$

$$\nabla^2\psi = 0, \;\; B = -\nabla\psi \tag{9}$$

that are <u>coupled</u> through equations (3) and (7) (with the definitions in Eqs. 5 and 8) would be a useful approach . These field solutions could include end effects, models of the primary strap, the resistive termination region, ion beam charges as a source for the electric potential, secondary electrons etc. Indeed, this may be quite close to the WARP code approach already being used by Dave and Enrique for all I know. The essential approximation would be the use of a continuous sheet model of the helix, in addition to the quasistatic approximation to Maxwell's equations.

An analytical model of wave propagation on the helix (not including end effects) can also be formulated with this model by going into Fourier space ($\exp(-jkz)$) and solving the Laplace equations subject to the usual boundary conditions. The result can be put in the form of transmission line equations with equivalent k-dependent capacitance and inductance per unit length. The magnetostatic field $\overrightarrow{H} = -\nabla\psi$ created by $K_\theta = nI$ identifies an equivalent inductance $L(k)$ using Eq (7), and a calculation of the electrostatic potential ϕ created by the line charge λ identifies an equivalent capacitance using $\lambda = C(k)\,\phi(r=a)$.

Traveling wave fields

In a forward moving traveling wave (beyond the excitation region) where everything is a function of $t - z/v_c$, we see from Eq. (7) that the voltage and magnetic flux have the same $f(t - z/v_c)$ dependence (are "in phase"), with peak values

$$V = n\,v_c\,\Phi \tag{10}$$

Therefore, the average axial magnetic field inside the helix (approximately constant in r at long wavelengths) is related to the voltage as

$$\langle B_z \rangle = \frac{V}{\pi a^2 n v_c} \tag{11}$$

For the oil helix in NDCX, a 9.5 kV peak voltage corresponds to a 100 gauss peak magnetic field.

The axial electric field at the helix, $E_z = -\partial V/\partial z$, is of course "90 degrees out of phase" with the voltage and axial magnetic field.

Speculations on secondary electron "cycling" mechanisms

The E and B fields of a traveling wave on the helix are sketched in the top figure that follows. The orbit of a secondary electron released near the peak of the axial electric field is sketched in the bottom figure. This electron would likely pick up appreciable velocity perpendicular to the magnetic field at the point shown, of order E/B. It would be accelerated along the B field lines by the electric field, but decelerated by the "magnetic mirror force". It is likely that this electron would cycle back and strike the insulator with appreciable energy, releasing more than one electron if the SEY is > 1.